

Why are losses from trade unlikely?

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Abstract

Examining a standard monopolistic competition model with unspecified utility/cost functions, we find *necessary and sufficient* conditions on their elasticities for welfare losses to arise from trade or market expansion. Two numerical examples explain the losses: excessive or insufficient entry of firms can be aggravated by market enlargement (under unrealistic elasticities).

JEL Codes: F12, L13

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Introduction

Gains from trade and large markets are an important issue in monopolistic competition theory (Melitz and Redding [7]), whereas possible *losses* are less studied, unlike in oligopoly settings (Brander and Krugman [1]). Trying to prove
5 the impossibility of harmful trade, we arrive instead at two counter-examples and a *criterion* (necessary and sufficient condition) for losses. The objective is to distinguish industries likely or unlikely to be harmed by globalization, by examining properties of their demand and supply functions.

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⁴In memoriam, 1973–2013.

This goal requires advanced modelling: variable elasticity of substitution
 10 (VES), unspecified preferences and general-form costs. Our setting deviates
 from Zhelobodko et al. [10] by allowing both convex and *concave* total cost. This
 generalization is needed for an important feature: indirect modelling of endoge-
 nous technology (R&D). Indeed, when R&D is possible, higher output fosters
 investment in marginal cost *reduction*, which implies concave cost (Bykadorov
 15 et al. [2]).

The main result is condition (9) on utilities/costs, necessary and sufficient
 for intra-sectoral trade gains or losses in a generalized Dixit-Stiglitz-Krugman
 model. In addition, two numerical examples demonstrate that this requirement
 is plausible, i.e., compatible with other reasonable properties of preferences and
 20 costs. Therefore both directions of market distortion shows as theoretically pos-
 sible: *excessive or insufficient entry can be aggravated by market growth*. How-
 ever, utilities/costs that satisfy (9) are uncommon, and the related discussion
 shows why trade losses are *unlikely* in the real economy.

1. Model

25 The model exposition follows Zhelobodko et al. [10] to ease comparison.
 Our closed economy exhibits monopolistic competition under *unspecified* addi-
 tive utility and cost functions, with variable marginal costs/elasticities. The only
 production factor is labor, supplied inelastically by L identical consumers/workers.
 A single sector involves an endogenous interval $[0, N]$ of identical firms produc-
 30 ing varieties, one variety per firm.

Each consumer maximizes utility in the form

$$U = \int_0^N u(x_i) di \rightarrow \max_{X \geq 0}, \text{ s.t. } \int_0^N p_i x_i di \leq 1. \quad (1)$$

Here $X = (x_j)_{j \leq N}$ is a function, $x_i \equiv x(i)$ denotes consumer's consumption of i -
 th variety, p_i is the price, $w \equiv 1$ is wage, index i everywhere replaces parentheses
 (i). As in Zhelobodko et al. [10], we use the elasticity operator $\mathcal{E}_g(z) \equiv \frac{zg'(z)}{g(z)}$
 defined for any function g , and the Arrow-Pratt concavity operator $r_g(z) \equiv$
 35 $-\frac{zg''(z)}{g'(z)} = -\mathcal{E}_{g'}(z)$.

For existence, uniqueness and symmetry of the equilibrium, we make the following weak restrictions on utility (Zhelobodko et al. [10], Mrzov and Neary [9]). At some zone $[0, \bar{z})$ of possible equilibria ($\bar{z} \leq \infty$), the elementary utility function $u(\cdot)$ is thrice differentiable—increasing ($u'(z) > 0$), strictly concave ($u''(z) < 0$), normalized ($u(0) = 0$)—and its main characteristics behave as
40 $r_u(z) \in [0, 1)$, $r_{u'}(z) < 2 \forall z \in [0, \bar{z})$.

Then the first-order condition (FOC) with a Lagrange multiplier λ entails the inverse demand function \mathbf{p} for any variety i :

$$\mathbf{p}(x_i, \lambda) = \frac{u'(x_i)}{\lambda}. \quad (2)$$

The marginal utility of income λ serves as the single market aggregate.

Each producer faces some total cost function $C(q)$ depending upon output $q \equiv Lx$, perceives function \mathbf{p} and λ as given, and maximizes profit

$$\pi(x, \lambda) \equiv \frac{u'(x)}{\lambda} xL - C(Lx) \rightarrow \max_{x \geq 0}.$$

(Here, choice of maximizers x , q or p brings an equivalent result, and the firm's index i is dropped by symmetry.) Denoting revenue $R(x, \lambda, L) \equiv u'(x) xL/\lambda$, we can formulate the FOC in usual terms of marginal revenue and marginal cost: $\frac{d}{dx} R(x, \lambda, L) - \frac{d}{dx} C(Lx) = 0$. The second-order condition (SOC) is

$$-\frac{d^2 \pi}{dx^2} = -\frac{d^2}{dx^2} R(x, \lambda, L) + \frac{d^2}{dx^2} C(Lx) > 0.$$

This assumption justifies symmetry of the equilibrium.

Equilibrium is a bundle $(\bar{x}, \bar{p}, \bar{\lambda}, \bar{N})$ satisfying the utility maximization condition (2); profit maximization FOC and SOC; free-entry and labor market clearing conditions:

$$R(\bar{x}, \bar{\lambda}, L) - C(L\bar{x}) = 0, \quad (3)$$

$$\bar{N}C(L\bar{x}) = L. \quad (4)$$

(Upper bar henceforth denotes equilibria.)

Now we can divide each producer's FOC by the free-entry condition to express our equilibrium through the elasticity of revenue \mathcal{E}_R , the elasticity of

inverse demand $\mathcal{E}_{\mathbf{p}}(x) \equiv \frac{x}{\mathbf{p}} \cdot \frac{\partial \mathbf{p}(x)}{\partial x} \equiv -r_u(x)$ and the cost elasticity $\mathcal{E}_C(q) \equiv \frac{q}{C} \cdot \frac{\partial C(q)}{\partial q}$:

$$\mathcal{E}_R(\bar{x}) \equiv 1 - r_u(\bar{x}) = \mathcal{E}_C(L\bar{x}). \quad (5)$$

45 The equilibrium consumption \bar{x} is determined here, whereas equilibrium prices \bar{p} and mass \bar{N} of firms can be found from the remaining equations. Therefore, each consumer's equilibrium welfare $\bar{U} = \bar{N}u(\bar{x})$ depends indirectly on market size L through the equilibrium magnitudes $\bar{x}(L), \bar{N}(L)$.

Totally differentiating the equilibrium equation (5) w.r.t. population size L and using (4), we express total utility elasticity $E_{\bar{U}/L}$ at equilibrium through other total elasticities $E_{\bar{N}/L} \equiv \frac{L}{\bar{N}} \cdot \frac{d\bar{N}}{dL}$, $E_{\bar{x}/L} \equiv \frac{L}{\bar{x}} \cdot \frac{d\bar{x}}{dL}$ and partial elasticity $\mathcal{E}_u \equiv \mathcal{E}_u(\bar{x}) \equiv \frac{z}{u(z)} \cdot \frac{\partial u(z)}{\partial z}$ as follows:

$$E_{\bar{U}/L} \equiv \frac{L}{\bar{U}} \cdot \frac{d\bar{U}}{dL} = E_{\bar{N}/L} + \mathcal{E}_u \cdot E_{\bar{x}/L}. \quad (6)$$

The SOC for profit maximization at equilibrium is

$$SOC \equiv r'_u(\bar{x}) \cdot \bar{x} + \mathcal{E}'_C(L\bar{x}) \cdot L\bar{x} > 0. \quad (7)$$

(Proofs are in Bykadorov et al. [3].)

50 2. Losses from market size

Lemma. *The local effect of a growing market on welfare can be expressed in elasticities (taken at the equilibrium values) as follows:*

$$E_{\bar{U}/L} = (1 - \mathcal{E}_u) - \frac{\bar{x}^2}{\mathcal{E}_u} \cdot \frac{\mathcal{E}'_u \cdot r'_u}{SOC} = r_u + \frac{L\bar{x}^2}{\mathcal{E}_u} \cdot \frac{\mathcal{E}'_u \cdot \mathcal{E}'_C}{SOC}. \quad (8)$$

This lemma enables us to establish the necessary and sufficient condition for “harmful trade” through the following claims, each highlighting some aspect of
55 market distortion.

Proposition. *Consider an equilibrium \bar{x} under market size L_0 . Any local welfare reduction caused by a growing market is equivalent to the following*

conditions on utility, revenue and cost elasticities:

$$E_{\bar{U}/L} < 0 \Leftrightarrow \mathcal{E}'_R(\bar{x}) < \mathcal{E}'_C(L_0\bar{x}) \cdot L_0 < \mathcal{E}'_R(\bar{x}) \cdot \frac{r_u(\bar{x})}{1 - \mathcal{E}_u(\bar{x})}. \quad (9)$$

For a more convenient interpretation, this double inequality can be reformulated as follows.

Corollary. (i) [Necessity]. *For any welfare reduction two conditions are necessary:*

$$\mathcal{E}'_u(\bar{x}) \cdot \mathcal{E}'_R(\bar{x}) < 0, \quad (10)$$

$$\mathcal{E}'_u(\bar{x}) \cdot \mathcal{E}'_C(L\bar{x}) < 0. \quad (11)$$

In particular, under convex cost ($\mathcal{E}'_C > 0$), such reduction requires both increasingly elastic revenue (IER) and decreasingly elastic utility (DEU).

(ii) [Sufficiency]. *For any utility satisfying inequality (10) at some \bar{x} under given L_0 , one can find a cost function C such that \bar{x} is an equilibrium, and welfare locally decreases w.r.t. L at L_0 . One can find also another cost function \tilde{C} that makes welfare locally increasing.*

Discussion. Under properties (9), (10) and (11) holding globally, these claims are easily extended from infinitesimal changes in population and welfare ($\frac{d\bar{U}}{dL}$) onto global ones ($\frac{\Delta\bar{U}}{\Delta L}$).

Why are equilibria satisfying all conditions (9), (10) and (11) *unlikely*? Property $\mathcal{E}'_R(x) \equiv -r'_u(x) < 0$ is called decreasingly elastic revenue (DER), being equivalent to increasingly elastic (strictly subconvex) demand (Mrzov and Neary [8]). The DER case is called realistic by Krugman [6] and subsequent papers (see Zhelobodko et al. [10]) because it generates decreasing prices under increasing competition; DER is perceived as “Marshall’s Second Law of Demand” by Mrzov and Neary [8]. Then, (9) becomes $1 > \frac{-\mathcal{E}'_C(L\bar{x}) \cdot L}{r'_u(\bar{x})} > \frac{r_u(\bar{x})}{1 - \mathcal{E}_u(\bar{x})}$. To get losses, some $C(\cdot)$ must fit this double inequality, compatible only when $\mathcal{E}_u < 1 - r_u = \mathcal{E}_R$, which is problematic. Indeed, finite $u'(0) > 0$ (choke-price) and negative $u'(\infty) < 0$ (satiabile demand) are reasonable assumptions that entail $\mathcal{E}_u = 1$, $\lim_{x \rightarrow \infty} \mathcal{E}_u(x) < 0$. Then, \mathcal{E}_u generally decreases:

$0 \geq \mathcal{E}_{\mathcal{E}_u} \equiv \mathcal{E}_R - \mathcal{E}_u : [0, \infty)$, except, maybe, for some interval. For $E_{\bar{U}/L} < 0$, the equilibrium must occur in this interval, which is empty for all typical utilities: CARA, HARA, CES, quadratic. Even for specially constructed exotic utilities, the interval is tiny, the freedom of choice in (9) being small: $\frac{r_u(\bar{x})}{1-\mathcal{E}_u(\bar{x})} \approx 1$ (see
85 our examples). Moreover, reasonable cost specifications $C \equiv f + c(f)q$ where function $c(f)$ decreases not too quickly, yield $\mathcal{E}'_C > 0$, which is incompatible with (9) when $\mathcal{E}'_R(x) < 0$. Summarizing, *too many very stringent conditions must hold simultaneously to generate losses from trade.*

Literature and interpretation. Elasticities' role in welfare is known since
90 Dixit and Stiglitz [4]: under linear cost ($C \equiv f + cq$), CES yields optimum, whereas firms' entry is socially excessive in DEU case ($\mathcal{E}'_u < 0$) but insufficient under IEU ($\mathcal{E}'_u > 0$). Under market expansion, such a distortion diminishes under DER, and the consumer enjoys a “double benefit” from growing variety and decreasing prices (Krugman [6]). This claim is generalized to convex cost
95 in Zhelobodko et al. [10], to costly trade in Mrzov and Neary [9], and to firms' heterogeneity in Dhingra and Morrow [5], where functions $r_u(x)$ and $1 - \mathcal{E}_u(x)$ are called “markup” and “social markup,” respectively. These two are “aligned” when both increase (DER, DEU) or both decrease (IER, IEU); when preferences are aligned, market expansion increases welfare. In economic terms, *“the market*
100 *maximizes markup but the social planner pursues social markup; when these goals are aligned, demand shifts alter private and social markups in the same direction”* (Dhingra and Morrow [5]).

We have shown that aligned preferences serve also as a *necessary* condition for welfare gains, in the sense of “gains occurring under any cost function.”
105 Another contribution is non-linear cost, e.g., endogenous technology. Though according to Dhingra and Morrow [5] “these are the demand-side elasticities that determine how resources are misallocated and when ... market expansion provides welfare gains” but (9) shows that costs also matter.

The only mathematical example of harmful trade in the New Trade theory
110 that we know—we have learned from Peter Neary in private talk, 2013. It

exploits a non-normalized utility, e.g., $u(x) = \sqrt{x} - a$ ($a > 0$), that violates our assumptions. However, we do not perceive violating normalization as legitimate. Indeed, an *arbitrary* constant added to the sub-utility should not change all the welfare conclusions from plus to minus. Instead, we provide two legitimate
115 examples.

[Insert Figure here]

Example 1: IER and DEU (excessive entry aggravated, Fig.2.1, left panel)

To demonstrate the possibility of harmful trade, we extend the CARA utility
120 with a linear or polynomial term:

$$u(x) = \begin{cases} 1 - \exp(-x) + 2x & \text{if } x < 2 \\ -\exp(-x) + 3x - 0.25x^2 & \text{if } x \geq 2 \end{cases}.$$

Here IER property holds under $x < 2$, where all our equilibria lie (interval $x \geq 2$ is constructed only to satisfy our restrictions on u). Elasticity $\mathcal{E}_u(x) = \frac{(2+\exp(-x))x}{1-\exp(-x)+2x}$ is computed and plotted by software as the thick orange dotted curve in the figure. Revenue elasticity $\mathcal{E}_R(x) = 1 - r_u(x) = \frac{\exp(-x)x}{2+\exp(-x)}$ is
125 painted thick blue. The initial market size is $L_1 = 3.4447$; related cost elasticity $\mathcal{E}_C(L_1x) = \frac{L_1cx}{f+L_1cx}$ is painted dashed thick magenta ($C(q) \equiv f + cq = 1 + q$). The equilibrium equation (5) here means the lower of the two intersections between $\mathcal{E}_R(x)$ and $\mathcal{E}_C(L_1x)$, almost indistinguishable because of *almost tangent* curves: $\frac{r_u(\bar{x})}{1-\mathcal{E}_u(\bar{x})} \approx 1 \Rightarrow \mathcal{E}'_C \approx \mathcal{E}'_R$. This (pink) equilibrium point is
130 $x_1 \approx 1.98683$, $U_1 \approx 2.12396$. When market size expands to $L_2 = 3.446$, we get a new, thin dashed, curve $\mathcal{E}_C(L_2x)$, which almost coincides with old curve $\mathcal{E}_C(L_1x)$. It brings a new (black) equilibrium point with *smaller* consumption and utility: $x_2 \approx 1.96165 < x_1$, $U_2 \approx 2.12389 < U_1$. The mass of firms increases ($N_1 = 0.439149 < N_2 = 0.444081$) but insufficiently.

135 Initially, excessive entry here implies inefficiently *high* average costs. Subsequently, an increase in L *pulls the average cost up, driving the economy further away from optimum. Individual consumption decreases, being insufficiently*

compensated by increasing variety. However, excessive entry is aggravated by market expansion only if cost elasticity is almost tangent to revenue elasticity, as one can understand from $\frac{r_u(\bar{x})}{1-\varepsilon_u(\bar{x})} \approx 1$.

Example 2: DER and IEU (insufficient entry aggravated, Fig.2.1, right panel)

Consider be-power utility

$$u(x) = \begin{cases} 0.5\sqrt{0.125+x} - 0.125x^{3/4} + 0.125 \cdot 2^{-1/4} & \text{if } x \geq 0.1 \\ 6.04076x - 28.4021x^2 & \text{if } x < 0.1 \end{cases}.$$

All our equilibria belong to $x \geq 0.1$; the initial interval $x \in [0, 0.1]$ is constructed to fit our assumption $u(0) = 0$. The non-linear cost function is $C(q) = 0.2 + 1.5 \exp\left(-1/\sqrt{q/7}\right)$. Fig.2.1-right maintains the same colours and legend as the left panel. Under $L_1 = 4.25$, the first equilibrium point is $x_1 \approx 0.669948$, $U_1 \approx 3.7994$. Increasing the market size to $L_2 = 4.47$, we get another equilibrium point with higher consumption $x_2 \approx 0.921192 > x_1$ and *smaller* utility $U_2 \approx 3.6732 < U_1$. The mass of firms decreases noticeably: $N_2 = 7.36122 < N_1 = 8.28935$.

Unlike the previous example, the initial social distortion takes here the form of insufficient entry (variety) and related *low* average costs. Then, in response to increasing L , *variety further decreases, being insufficiently compensated by the consumption hike* and thus further departs from optimum. Again, distortion is aggravated by market expansion *only* when cost elasticity is almost tangent to revenue elasticity (8).

To summarize, *losses from trade require several highly stringent conditions to hold simultaneously: increasingly elastic misaligned preferences and quite specific costs*. For more definite conclusion, condition (9) should be falsified or confirmed empirically.

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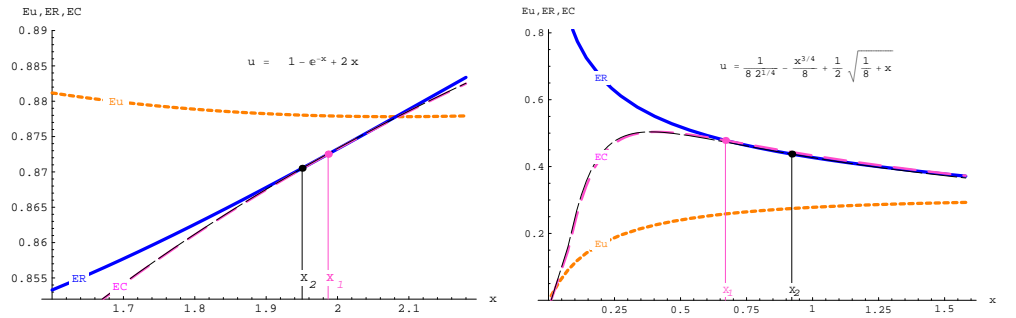


Figure 0.1: Misaligned elasticities and welfare loss: two examples